Nama : Behvi Efrian Emirsan
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Tugas Online 5
Matematika 2

I. $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 3x^2 + 7$

1. Dicari penyelesaian karakteristik / homogen

$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 0$

gunakan simbol $D = \frac{d}{dx}$

$D^2y + 6Dy = 0$

$(D^2 + 6D)\,y = 0$

$D^2 + 6D = 0$

$D(D+6) = 0$

$D_1 = 0 \qquad D_2 = -6$

penyelesaian homogen : $y = C_1e^0 + C_2e^{-6x}$

$y = C_1 + C_2e^{-6x}$

2. Dicari penyelesaian partikular / lengkap

$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 3x^2 + 7$

$y = Ax^2 + Bx + C$

$\frac{dy}{dx} = 2Ax + B$

$\frac{d^2y}{dx^2} = 2A$

$= 2A + 6(2Ax + B) = 3x^2 + 7$

Koefisien : $x \Rightarrow 12A = 0$

$A = 0$

Koefisien konstan $\Rightarrow 2A + 6B = 7$

$B = \frac{7}{6}$

Penyelesaian lengkap :

$y = Ax^2 + Bx + C$

$y = \frac{7}{6}x$

Penyelesaian umum :

$y = C_1 + C_2 e^{-6x} + \frac{7}{6}x$

II. $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 9y = 2\sin 2x$

1. Dicari penyelesaian karakteristik / homogen

$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 9y = 0$

gunakan simbol $D = \frac{d}{dx}$

$D^2y - 3Dy + 9y = 0$

$(D^2 - 3D + 9)y = 0$

$D^2 - 3D + 9 = 0$

$D_{1,2} = \frac{3 \pm \sqrt{9 - 4 \cdot 1 \cdot 9}}{2 \cdot 1} = \frac{3 \pm \sqrt{-27}}{2} = \frac{3}{2} \pm \sqrt{\frac{27}{2}}\sqrt{-1}$

$D_1 = \frac{3}{2}$, $D_2 = \sqrt{\frac{27}{2}}$

penyelesaian homogen : $y = e^{3/2x}\left\{C_1 \cos\sqrt{\frac{27}{2}}x + C_2 \sin\sqrt{\frac{27}{2}}x\right.$

2. Dicari penyelesaian partikular / lengkap

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 9y = 2\sin 2x$$

$$y = A\cos 2x + B\sin 2x$$

$$\frac{dy}{dx} = -2A\sin 2x + 2B\cos 2x$$

$$\frac{d^2y}{dx^2} = -4A\cos 2x - 4B\sin 2x$$

$$= (-4A\cos 2x - 4B\sin 2x) - 3(-2A\sin 2x + 2B\cos 2x) + 9(A\cos 2x + B\sin 2x) = 2\sin 2x$$

Koefisien : $\cos 2x \Rightarrow -4A - 6B + 9A = 0$

$$5A - 6B = 0$$

$$A = \frac{6}{5}B \Rightarrow A = \frac{60}{305}$$

$\sin 2x \Rightarrow -4B + 6A + 9B = 2$

$$6A + 5B = 2$$

$$\frac{36}{5}B + 5B = 2$$

$$\frac{61}{5}B = 2$$

$$B = \frac{10}{61}$$

Penyelesaian lengkap :

$$y = \frac{60}{305}\cos 2x + \frac{10}{61}\sin 2x$$

Penyelesaian umum :

$$y = e^{3/2x}\left\{C_1\cos\sqrt{\frac{27}{2}}x + C_2\sin\sqrt{\frac{27}{2}}x\right. + \frac{60}{305}\cos 2x + \frac{10}{61}\sin 2x$$

III. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 3e^{2x} + \cos 2x$

1. Dicari penyelesaian karakteristik / homogen

$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$

gunakan simbol $D = \frac{d}{dx}$

$D^2y + 5Dy + 6y = 0$

$(D^2 + 5D + 6)\,y = 0$

$D^2 + 5D + 6 = 0$

$(D + 2)(D + 3) = 0$

$D_1 = -2 \quad , D_2 = -3$

penyelesaian homogen : $y = C_1e^{-2x} + C_2e^{-3x}$

2. Dicari penyelesaian partikular / lenghap

$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 3e^{2x} + \cos 2x$

$y = Ae^{2x} + B\cos 2x + C\sin 2x$

$\frac{dy}{dx} = 2Ae^{2x} - 2B\sin 2x + 2C\cos 2x$

$\frac{d^2y}{dx^2} = 4Ae^{2x} - 4B\cos 2x - 4C\sin 2x$

$= (4Ae^{2x} - 4B\cos 2x - 4C\sin 2x) + 5(2Ae^{2x} - 2B\sin 2x + 2C\cos 2x) + 6(Ae^{2x} + B\cos 2x + C\sin 2x) = 3e^{2x} + \cos 2x$

Koefisien : $e^{2x} \Rightarrow 4A + 10A + 6A = 3$

$20A = 3$

$A = \frac{3}{20}$

$\cos 2x \Rightarrow -4B + 10C + 6B = 1$

$2B + 10C = 1$

$2B + 10(5B) = 1$

$52B = 1$

$B = \frac{1}{52}$

$\sin 2x \Rightarrow \quad -4C - 10B + 6C = 0$

$2C - 10B = 0$

$2C = 10B$

$C = 5B$

$C = 5\left(\frac{1}{52}\right)$

$C = \frac{5}{52}$

Penyelesaian lengkap :

$$y = \frac{3}{20} e^{2x} + \frac{1}{52} \cos 2x + \frac{5}{52} \sin 2x$$

Penyelesaian umum :

$$y = C_1 e^{-2x} + C_2 e^{-3x} + \frac{3}{20} e^{2x} + \frac{1}{52} \cos 2x + \frac{5}{52} \sin 2x$$

IV. $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} - y = \cos 2x + 5 \sin x$

1. Dicari penyelesaian karakteristik / homogen

$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} - y = 0$

gunakan simbol $D = \frac{d}{dx}$

$D^2y + 6Dy - y = 0$

$(D^2 + 6D - 1)y = 0$

$D^2 + 6D - 1 = 0$

$$D_{1,2} = \frac{-6 \pm \sqrt{36 - 4 \cdot 1 \cdot (-1)}}{2}$$

$$= \frac{-6 \pm \sqrt{40}}{2} = -\frac{6}{2} \pm \frac{\sqrt{40}}{2}$$

$D_1 = -\frac{6}{2}$, $D_2 = \frac{\sqrt{40}}{2}$

Penyelesaian homogen : $y = e^{-3x}\left\{C_1 \cos \frac{\sqrt{40}}{2} x + C_2 \sin \frac{\sqrt{40}}{2} x\right\}$

2. Dicari penyelesaian partikular / lengkap

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} - y = \cos 2x + 5\sin x$$

$$y = A\cos 2x + B\sin 2x + C\cos x + D\sin x$$

$$\frac{dy}{dx} = -2A\sin 2x + 2B\cos 2x - C\sin x + D\cos x$$

$$\frac{d^2y}{dx^2} = -4A\cos 2x - 4B\sin 2x - C\cos x - D\sin x$$

$$= (-4A\cos 2x - 4B\sin 2x - C\cos x - D\sin x) + 6(-2A\sin 2x + 2B\cos 2x - C\sin x + D\cos x) - (A\cos 2x + B\sin 2x + C\cos x + D\sin x) = \cos 2x + 5\sin x$$

koefisien : $\cos 2x \Rightarrow -4A + 12B - A = 1$

$$12B - 5A = 1$$

$$12B - 5\left(-\frac{5}{12}B\right) = 1$$

$$12B + \frac{25}{12}B = 1$$

$$\frac{169}{12}B = 1$$

$$B = \frac{12}{169}$$

$\sin 2x \Rightarrow -4B - 12A - B = 0$

$$-12A - 5B = 0$$

$$-12A = 5B$$

$$A = -\frac{5}{12}B$$

$$A = -\frac{5}{12}\left(\frac{12}{169}\right)$$

$$A = -\frac{5}{169}$$

$\cos x \Rightarrow -C + 6D - C = 0$

$$6D - 2C = 0$$

$$6D = 2C$$

$$D = \frac{1}{3}C$$

$$D = \frac{1}{3}\cdot\left(-\frac{15}{20}\right)$$

$$D = -\frac{1}{4}$$

$\sin x \Rightarrow -D - 6C - D = 5$

$-6C - 2D = 5$

$-6C - \frac{2}{3}C = 5$

$-\frac{20}{3}C = 5$

$C = -\frac{15}{20}$

Penyelesaian lengkap :

$$y = -\frac{5}{169}\cos 2x + \frac{12}{169}\sin 2x - \frac{15}{20}\cos x - \frac{1}{1}\sin x$$

Penyelesaian umum :

$$y = e^{-3x}\left[C_1 \cos \frac{\sqrt{10}}{2}x + C_2 \sin \frac{\sqrt{10}}{2}x\right] - \frac{5}{169}\cos 2x + \frac{12}{169}\sin 2x - \frac{15}{20}\cos x - \frac{1}{1}\sin x$$

V. $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 2y = 1x^2 + 5x + 1 + 2e^{3x}$

1. Dicari penyelesaian karakteristik / homogen

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 2y = 0$$

gunakan simbol $D = \frac{d}{dx}$

$D^2y + 6Dy + 2y = 0$

$(D^2 + 6D + 2)y = 0$

$D^2 + 6D + 2 = 0$

$D_{1,2} = \frac{-6 \pm \sqrt{28}}{2} = -\frac{6}{2} \pm \frac{\sqrt{28}}{2}$

$D_1 = -\frac{6}{2} = -3, \quad D_2 = \frac{\sqrt{28}}{2}$

penyelesaian homogen : $y = e^{-3x}\left[C_1 \cos \frac{\sqrt{28}}{2}x + C_2 \sin \frac{\sqrt{28}}{2}x\right]$

2. Dicari penyelesaian partikular / lengkap

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 2y = 4x^2 + 5x + 1 + 2e^{3x}$$

$$y = Ax^2 + Bx + C + De^{3x}$$

$$\frac{dy}{dx} = 2Ax + B + 3De^{3x}$$

$$\frac{d^2y}{dx^2} = 2A + 9De^{3x}$$

$$= (2A + 9De^{3x}) + 6(2Ax + B + 3De^{3x}) + 2(Ax^2 + Bx + C + De^{3x}) = 4x^2 + 5x + 1 + 2e^{3x}$$

Koefisien : $x^2 \Rightarrow 2A = 4$

$A = 2$

$x \Rightarrow 12A + 2B = 5$

$12(2) + 2B = 5$

$2B = -19$

$B = -\frac{19}{2}$

$e^{3x} \Rightarrow 9D + 18D + 2D = 2$

$29D = 2$

$D = \frac{2}{29}$

Koefisien konstan : $2A + 6B + 2C = 1$

$4 - 57 + 2C = 1$

$2C = 54$

$C = 27$

Penyelesaian lengkap :

$$y = 2x^2 - \frac{19}{2}x + 27 + \frac{2}{29}e^{3x}$$

Penyelesaian umum :

$$y = e^{-3x}\left\{C_1 \cos \frac{\sqrt{28}}{2}x + C_2 \sin \frac{\sqrt{28}}{2}x\right\} + 2x^2 - \frac{19}{2}x + 27 + \frac{2}{29}e^{3x}$$