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Tugas Online 4

Matematika 2

1. $\frac{dy}{dx} + 2xy = 7x$

$$y = e^{-\int P(x)\,dx} \left\{ \int Q(x) \cdot e^{\int P(x)\,dx}\, dx + c \right\}$$

$$y = e^{-\int 2x\,dx} \left\{ \int 7x \cdot e^{\int 2x\,dx}\, dx + c \right]$$

$$y = e^{-x^2} \left\{ \int 7x \cdot e^{x^2}\, dx + c \right\}$$

Misal : $u = x^2$ $\qquad dx = \frac{du}{2x}$

$\frac{du}{dx} = 2x$

$$y = e^{-x^2} \left\{ \int 7\cancel{x} \cdot e^{u} \frac{du}{2\cancel{x}} + c \right]$$

$$y = e^{-x^2} \left\{ \frac{7}{2} \int e^{u}\, du + c \right\}$$

$$y = e^{-x^2} \left\{ \frac{7}{2} e^{u} + c \right\}$$

$$y = e^{-x^2} \left[\frac{7}{2} e^{x^2} + c \right\}$$

2. $(2x + y)\,dy + (x - 3y)\,dx = 0$

$(2x + y)\,dy + (x - 3y)\,dx = 0 \quad | : y$

$\left(2\frac{x}{y} + 1\right) dy + \left(\frac{x}{y} - 3\right) dx = 0$

Misal : $u = \frac{x}{y}$

$x = u \cdot y$

$dx = dy \cdot u + y \cdot du$

$= (2u + 1)\,dy + (u - 3)\,\{dy \cdot u + y \cdot du\} = 0$

$= (2u + 1)\,dy + (u^2 - 3u)\,dy + (u - 3)\,y \cdot du = 0$

$= (u^2 - u + 1)\,dy + (u - 3)\,y \cdot du = 0 \quad | : y$

$= (u^2 - u + 1)\,\frac{dy}{y} + (u - 3)\,du = 0 \quad | : (u^2 - u + 1)$

$= \frac{dy}{y} + \frac{(u - 3)}{(u^2 - u + 1)}\,du = 0$

$= \int \frac{dy}{y} + \frac{(u - 3)}{(u^2 - u + 1)}\,du = 0$

$= \ln y + \int \frac{1/2\,(2u - 1) - 5/2}{(u^2 - u + 1)}\,du = 0$

$= \ln y + \frac{1}{2} \int \frac{2u - 1}{u^2 - u + 1}\,du - \frac{5}{2} \int \frac{1}{u^2 - u + 1}\,du = 0$

$= \ln y + \frac{1}{2} \ln |u^2 - u + 1| - \frac{5}{2} \int \frac{1}{(u - \frac{1}{2})^2 + \frac{3}{4}}\,du = 0$

$= \ln y + \frac{1}{2} \ln |u^2 - u + 1| - \frac{5}{2} \left[\frac{1}{\sqrt{\frac{3}{4}}} \operatorname{arc\,tg} \frac{(u - \frac{1}{2})^2}{\sqrt{\frac{3}{4}}} \right]$

$= \ln y + \frac{1}{2} \ln \left| \frac{x^2}{y^2} - \frac{x}{y} + 1 \right| - \frac{5}{2} \left\{ \frac{1}{\sqrt{\frac{3}{4}}} \operatorname{arc\,tg} \frac{(x/y - \frac{1}{2})^2}{\sqrt{\frac{3}{4}}} \right\}$

3. $3x\,(y^5+9)\,dy + 6x^5 y\,dx = 0$

$= 3x\,(y^5+9)\,dy + 6x^5 y\,dx = 0 \quad | : x$

$= 3\,(y^5+9)\,dy + 6x^4 y\,dx = 0 \quad | : y$

$= 3\,\frac{(y^5+9)}{y}\,dy + 6x^4\,dx = 0$

$= 3\int \frac{y^5}{y}\,dy + 27\int \frac{1}{y}\,dy + \int 6x^4\,dx = 0$

$= 3\left\{\frac{1}{5}y^5\right\} + 27\ln y + \frac{6}{5}x^5 = C$

$= \frac{3}{5}y^5 + 27\ln y + \frac{6}{5}x^5 = C$

4. $(5x^3 + y^3)\,dy + 6x^2 y\,dx = 0$

$= (5x^3 + y^3)\,dy + 6x^2 y\,dx = 0 \quad | : y^3$

$= \left(5\frac{x^3}{y^3} + 1\right)dy + 6\frac{x^2}{y^2}\,dx = 0$

Misal : $u = \frac{x}{y}$ $\qquad dx = dy.\,u + y.\,du$

$x = y.\,u$

$= (5u^3+1)\,dy + 6u^2\{dy.\,u + y.\,du\} = 0$

$= (5u^3+1)\,dy + 6u^3 dy + 6u^2 y.\,du = 0$

$= (11u^3+1)\,dy + 6u^2 y.\,du = 0$

$= (11u^3+1)\,dy + 6u^2 y.\,du = 0 \quad | : y$

$= (11u^3+1)\,\frac{dy}{y} + 6u^2.\,du = 0 \quad | : (11u^3+1)$

$= \frac{dy}{y} + \frac{6u^2}{11u^3+1}\,du = 0$

$= \int\frac{dy}{y} + \int\frac{6u^2}{11u^3+1}\,du = 0$

$= \ln y + \int\frac{6/33\,(33u^2)}{11u^3+1}\,du = 0$

$= \ln y + \frac{6}{33}\ln|11u^3+1| = C$

$= \ln y + \frac{6}{33}\ln\left|11\frac{x^3}{y^3} + 1\right| = C$

6. $(9x - 15y^2)\,dx + 12xy\,dy = 0$

$= (9x - 15y^2)\,dx + 12xy\,dy = 0 \quad | : x^2$

$= \left(9 - 15\frac{y^2}{x^2}\right)dx + 12\frac{y}{x}\,dy = 0$

Misal : $u = \frac{y}{x}$ $\qquad dy = dx \cdot u + x \cdot du$.

$y = u \cdot x$

$= (9 - 15u^2)\,dx + 12u\,dy = 0$

$= (9 - 15u^2)\,dx + 12u\{dx \cdot u + x \cdot du\} = 0$

$= (9 - 15u^2)\,dx + 12u^2\,dx + 12u\,x\,du = 0$

$= (9 - 3u^2)\,dx + 12u\,x\,du = 0 \quad | : x$

$= (9 - 3u^2)\frac{dx}{x} + 12u\,du = 0 \quad | : (9 - 3u^2)$

$= \frac{dx}{x} + \frac{12u}{9 - 3u^2}\,du = 0$

$= \int \frac{dx}{x} + \int \frac{12u}{9 - 3u^2}\,du = 0$

$= \ln x - 2\int \frac{-6u}{9 - 3u^2}\,du = 0$

$= \ln x - 2\ln|9 - 3u^2| = C$

$= \ln x - 2\ln\left|9 - 3\frac{y^2}{x^2}\right| = C.$